Long-run Marginal Cost Pricing Based on Analytical Method for Revenue Reconciliation

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***Abstract*-- Incremental and marginal approaches are two different types of methods to price the use of networks. The major difference between them is in the way they evaluate the costs imposed by network users. The former calculates network charges through simulation and the latter derives charges with a sensitivity-based analytical approach. Both charging models aim to send cost-reflective economic signals to customers, providing an economic climate for the cost-effective development of networks.**

In this paper, a novel long-run marginal cost (LRMC) pricing methodology based on analytical method is proposed to reflect the impacts on the long-run costs imposed by a nodal injection through sensitivity analysis. The sensitivity analysis consists of three partial differentiations: i) the sensitivity of circuit power flow with respect to nodal power increment, ii) the sensitivity of the time to reinforce network with respect to changes in circuit power flows, and iii) the sensitivity of present value of future reinforcement with respect to changes in time to reinforce. Two test systems are employed to illustrate the principles and implementation of the proposed method. Results from incremental and marginal approaches under different system conditions are compared and contrasted in terms of charges and tariffs. The proposed method, as demonstrated in the test systems, can produce forward-looking charges that reflect the extent of network utilization levels in addition to the distance that power must travel from points of generation to points of consumption. Furthermore, the proposed method is able to provide further insights into factors influencing network charges.

***Index Terms*-- Long-run marginal cost, Long-run incremental cost, Network charging, Load growth rate**

I. Introduction

NETWORK charges are charges against network users for their use of a network. Methodologies used for setting network charges need to recover the costs of capital, operation and maintenance of a network and provide forward-looking, economically efficient messages for both consumers and generators [1, 2]. In order to achieve these objectives, it is essential that network charges can reflect the costs/benefits that new network users impose on networks. It is for this reason that the concept of incremental/marginal charging methodologies is introduced to reflect the costs of network operation and development incurred by new generation and load connection [1, 3, 4].

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Developing a long-run pricing model has been viewed as a formidable task. Previously proposed methodologies fall into two categories: long-run incremental cost pricing and long-run marginal cost pricing [1, 5-7]. The biggest difference between them is in the way they evaluate the effects on the long-term network development costs from a nodal injection. The long-run incremental charge for a nodal is evaluated by comparing the present value of future reinforcement with and without the nodal injection. This type of charging methodology is fairly easy to implement but takes long computational time for a large- system. On the other hand, marginal methods use analytical equations to evaluate the impact of nodal injection on long-run network development costs [1, 8]. This type of methodology is computationally efficient but based on the assumption that the relationship resulted from a small injection/withdrawal can be extrapolated to large injection/withdrawal. Inaccuracies will be resulted in as the relationship between the nodal injection and the network development costs is highly non-linear.

There are some papers focusing on the difference and relationship between the two type pricing [8, 9] and the use of these charging methods in real networks [10-13]. However, most of them require a least-cost network planning to determine the changes in network development costs from nodal generation/demand increment; but the knowledge of the future generation/demand is far from certain. Furthermore, these methods passively react to a set of projected future generation/demand patterns, not able to provide financial incentives to guide new network users to appropriate locations that lead to the least network development costs[14].

The first method that directly links long-term network development costs with nodal increment was presented by Li and Tolley [15]. The proposed long-run incremental cost (LRIC) pricing makes use of the un-used capacity of an exiting network to reflect the costs of advancing or deferring future investment consequent upon the addition of generation or load at each study node. For LRIC charges for each node, two load flow runs are required to assess if the nodal increment brings forward or defers the future reinforcement. Such simulation approach is easy to implement and can provide forward-looking signals to reflect the extent of the use of the network by a new connectee. The shortcoming is that the simulation approach takes much longer time to calculate charges for large systems, as the computational time rises exponentially with the increasing size of systems. Further, it can be difficult to detect implementation errors with the simulation approach.

In this paper, a novel long-run marginal cost (LRMC) charging method is proposed following the same principle of [15], but utilizing sensitivity analysis to significantly reduce the computational burden for large systems. In the proposed LRMC approach, the change of present value of future reinforcement with respect to a nodal power increment is represented by three partial differentiations: i) sensitivity of circuit loading level with regard to nodal injection, ii) sensitivity of time to reinforce with respect to circuit loading level, and iii) sensitivity of the present value of future reinforcement with respect to time to reinforce. Using the sensitivity approach, LRMC calculates charges that can reflect very small changes in nodal generation/demand accurately compared with LRIC model. In practice, however, the nodal increment can be large, and therefore LRMC might introduce inaccuracy for larger increment compared with the LRIC approach, as the latter can accurately simulate the change in network loading conditions incurred by a large nodal increment. Two test systems are employed to compare the proposed LRMC approach with LRIC method under different load growth rates (LGRs), different loading levels and with different sizes of injections for LRIC. The comparison shows the boundary conditions in which the two methods conform well, and in which the two depart and LRMC is no longer appropriate to be applied. Further, in order to compare the economical signals provided by the two charging models to network users, tariffs reconciled from the LRIC and LRMC charges with two reconciliation methods are also discussed.

The rest of the paper is organized as follows: section II gives a brief introduction to LRIC charging approach. In section III, the novel LRMC charging method is presented. Section IV introduces two commonly used scaling methods for revenue reconciliation. Section V provides two test systems to compare the results derived from LRIC and LRMC. Section VI provides some discussions concerning the proposed method. Finally, some conclusions are drawn in section VII.

II. Long-Run Incremental Cost Pricing Model

In the original LRIC pricing model [15], for components in network that are affected by a nodal injection, there will be a cost associated for it if the investment is accelerated or a credit if it is deferred. The LRIC model has the following three implementation steps.

A. Present Value of Future Investment

If a circuit l has a maximum allowed power flow of C_l, supporting a power flow of P_l, the number of years it takes P_l to grow to C_l under a given LGR, r, can be determined with

$$C_l = P_l \cdot (1+r)^{n_l} \tag{1}$$

Where, n_l is the number of years taking P_l to reach C_l.

Rearranging (1) and taking the logarithm of it gives

$$n_l = \frac{\log C_l - \log P_l}{\log(1+r)} \tag{2}$$

Assume that investment will occur in year n_l when the circuit utilization reaches C_l and with a chosen discount rate of d, the present value of future investment is

$$PV_l = \frac{Asset_l}{(1+d)^{n_l}} \tag{3}$$

Where, $Asset_l$ is the modern equivalent asset cost.

B. Cost Associated with Power Increment

If power flow change along line l is ΔP_l as a result of a nodal injection, the time to future reinforcement will change from year n_l to year n_{lnew}, defined by

$$C_l = (P_l + \Delta P_l) \cdot (1+r)^{n_{lnew}} \tag{4}$$

Equation (4) gives the new investment horizon n_{lnew}

$$n_{lnew} = \frac{\log C_l - \log(P_l + \Delta P_l)}{\log(1+r)} \tag{5}$$

The new present value of future reinforcement becomes,

$$PV_{lnew} = \frac{Asset_l}{(1+d)^{n_{lnew}}} \tag{6}$$

The change in present value as a result of the injection is given by

$$g(r) = \Delta PV_l = Asset_l \cdot \left(\frac{1}{(1+d)^{n_{lnew}}} - \frac{1}{(1+d)^{n_l}} \right) \tag{7}$$

The incremental cost for circuit l is the annuitized change in present value of future investment over its life span,

$$\Delta IC_l = \Delta PV_l \cdot AnnuityFactor \tag{8}$$

C. Long-run Incremental Cost

The nodal LRIC charge is the summation of incremental cost over all circuits supporting it, given by

$$LRIC_i = \frac{\sum_l \Delta IC_l}{\Delta PI_i} \tag{9}$$

Where, ΔPI_i is the size of power injection at node i, and here we assign it to be 1MW.

In practice, all networks are designed to withstand credible contingencies, but this comes at a significant cost to network development. For the LRIC pricing model, it is important to recognise the level of spare capacity that is reserved for catering N-1 contingency. This can be determined by conducting a full N-1 contingency analysis. For each circuit, the base power flow and the maximum contingency flow are determined from base power flow and contingency analysis. Here, contingency factor is defined as the ratio of the maximum contingency flow over the circuit's base flow [16]. The maximum allowed power flow each circuit can carry considering N-1 contingency is

$$C_l = \frac{Rated\ Capacity_l}{Contingency\ Factor_l} \tag{10}$$

III. Long-Run Marginal Cost Pricing Model

The core of the LRIC method is to reflect: i) how a nodal injection might affect the level of spare capacity of network assets that support this injection, ii) how the change in spare capacity would influence the time to reinforce these assets, iii) how the change in time to reinforce can impact the present value of future reinforcement. These impacts can be approximated through three-step partial differentiations, which form the core of LRMC, given as

$$\frac{\partial PV_l}{\partial PI_n} = \frac{\partial PV_l}{\partial n_l} \cdot \frac{\partial n_l}{\partial P_l} \cdot \frac{\partial P_l}{\partial PI_n} \quad (11)$$

Where, P_l is the power flow along circuit l linking nodes i and j, n_l is the time to reinforce circuit l and PV_l is the present value of future reinforcement cost for circuit l.

Mathematically, the LRMC pricing can be implemented through the following steps.

A. Sensitivity of Circuit Power Flow to Nodal Injection

Equation (12) represents active power flow along a circuit from bus i to bus j.

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \quad (12)$$

If there is a small injection PI_n at node n, the effect on P_{ij} can be obtained by

$$\frac{\partial P_{ij}}{\partial PI_n} = \frac{\partial P_{ij}}{\partial V_i}\frac{\partial V_i}{\partial PI_n} + \frac{\partial P_{ij}}{\partial V_j}\frac{\partial V_j}{\partial PI_n} + \frac{\partial P_{ij}}{\partial \theta_i}\frac{\partial \theta_i}{\partial PI_n} + \frac{\partial P_{ij}}{\partial \theta_j}\frac{\partial \theta_j}{\partial PI_n} \quad (13)$$

Where, $\frac{\partial P_{ij}}{\partial V_i}$, $\frac{\partial P_{ij}}{\partial V_i}$, $\frac{\partial P_{ij}}{\partial \theta_i}$, and $\frac{\partial P_{ij}}{\partial \theta_j}$ can be calculated from (12) by calculating its partial derivates with regard to V_i, V_j, θ_i, θ_j.

In order to obtain the remaining parts in (13), sensitivity analysis is employed in (14) to represent the relationships between a change in nodal power and changes in voltage magnitudes and angles. Jacobian matrix in (14) is the one obtained in the last iteration of power flow analysis.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix} \cdot \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = [J] \cdot \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (14)$$

By applying (12) - (14), the effects of power injection at a node on circuits' power flows can be evaluated.

B. Sensitivity of Time to Reinforce to Circuit Power Flow

From (2), taking derivate of the time to reinforce with respect to circuit power flow gives

$$\frac{\partial n_l}{\partial P_l} = -\frac{1}{P_l \cdot \log(1+r)} \quad (15)$$

For a fixed LGR, the only factor that influences the sensitivity of time to reinforce to the power flow along a circuit is the circuit's loading level. The sensitivity of time to reinforce to the circuit's power flow can be either positive or negative. The negative sign implies that an increase in loading level reduces or brings forward time to reinforce and, a decrease in loading level increases or defers time to reinforce.

C. Sensitivity of Present Value of Future Reinforcement to Time to Reinforce

Similarly, from (3), taking derivative of PV_l with respect to n_l gives

$$\frac{\partial PV_l}{\partial n_l} = -\frac{Asset_l \cdot \log(1+d)}{(1+d)^{n_l}} \quad (16)$$

This formula represents how the change of time to reinforce affects the present value of future reinforcement. Here, because both asset cost and discount rate are fixed, the only factor influencing the level of sensitivity is time to reinforce. The negative sign indicates that a rise in time to reinforce lowers the present value of future reinforcement and, a fall in time to reinforce increases it.

D. Sensitivity of Present Value of Future Reinforcement to Nodal Injection

Combining (13), (15) and (16) into (17) and replacing n_l with (2) leads to the sensitivity of the present value of future reinforcement for a circuit to a nodal injection at node n

$$\frac{\partial PV_l}{\partial PI_n} = \frac{Asset_l}{P_l} \cdot \frac{\log(1+d)}{\log(1+r)} \cdot \left(\frac{P_l}{C_l}\right)^{\frac{\log(1+d)}{\log(1+r)}} \cdot \frac{\partial P_l}{\partial PI_n} \quad (17)$$

Where, $\frac{\partial P_l}{\partial PI_n}$ is from (13).

As can be seen from (17), for a circuit supporting the nodal injection at bus n, its cost, LGR, and the chosen discount rate are fixed. The factors that influence the change in the present value of future reinforcement as a result of the nodal injection are the circuit's loading level, the sensitivity of circuits' loading levels to the nodal injection, and the time to reinforce. For circuits with low sensitivities to the nodal injection, even if they are heavily loaded, they will still have a low LRMC charge for the node, as the nodal injection causes very little change to the time to reinforce. On the other hand, even for lightly loaded circuits, if their sensitivities to the nodal injection are high, they will see larger LRMC charges for the node as the nodal injection triggers big change in time to reinforce. The chosen LGR is another factor affecting the calculated LRMC charges, a low LGR can lead to high charges and a high LGR can result in low charges, depending on the level of the circuit's utilization.

E. Long-run Marginal Cost

The LRMC charge for node n is the sum of LRMC charges over all circuits that support the nodal injection, multiplied by an annuity factor. The charge is given by,

$$LRMC_n = \sum_l \frac{\partial PV_l}{\partial PI_n} \cdot AnnuityFactor \quad (18)$$

IV. Revenue Reconciliation

It should be noted that neither incremental nor marginal charges may be able to recover the revenue allowed for Distribution Network Operators (DNOs). Revenue reconciliation process is therefore generally required to adjust the nodal incremental or marginal prices so that the revenue recovered from network charges can meet the target revenue. The mechanisms used by DNOs are equally important due to the fact that in practice, a large proportion of their revenue may be recovered through such scaling mechanism and it may have a significant impact on the relative level of nodal tariffs.

There are two commonly adopted revenue reconciliation approaches to adjust the nodal prices, namely "fixed adder" and "fixed multiplier"[17]. The fixed adder method adds/subtracts a constant amount to/from the nodal charges to make up for the revenue shortfall/surplus. The multiplier method scales the nodal charges by a constant factor corresponding to the ratio of the target revenue to the recovered revenue. Equations (19) and (20) describe how they adjust nodal LRIC or LRMC charges.

$$tariff_i = Charge_i + adder \tag{19}$$

$$tariff_i = Charge_i \cdot (1 + multiplier) \tag{20}$$

In the following section, the two methods are used to examine how LRIC and LRMC models affect the tariffs.

V. Example Demonstration

A. Two-Busbar Test System Demonstration

The comparison of the two long-run charging methods is firstly carried out on a simple network shown in Fig. 1. Suppose that the rating of L_f is 45MW after security redundancy and its cost is £3,193,400. Taking 6.9% discount rate and 40 years life span leads to its annuity cost as £236,760/yr.

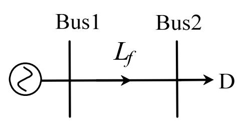
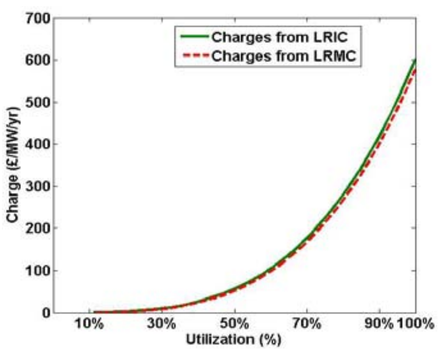
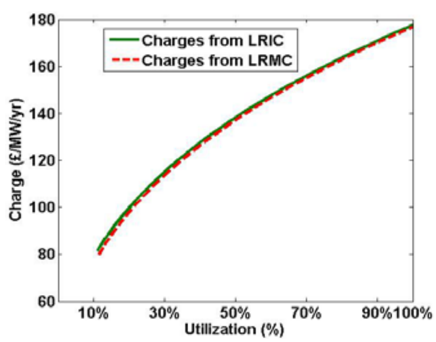


Fig. 1. Layout of two-circuit test system

As expected, if LRIC charges are calculated with a small injection - 0.1MW, LRMC yields similar results with LRIC in both low and high LGR cases and at both low and high circuit loading levels.



(a) 1.5% load growth rate case



(b) 5%load growth rate case

Fig. 2. Charge comparison with 1MW injection for LRIC

Fig. 2 compares the results with 1MW nodal injection for LRIC under two underlying growth rates, 1.5% and 5%. Generally, they are quite close at the most loading levels, with few exceptions. In the small LGR case, the difference in charges from the two methods grows with the increasing circuit's utilization. In the high LGR case, the charge difference decreases with the increase of loading level.

The apparent difference in charges is due to the different calculation concepts of the two approaches, demonstrated in Fig.3. LRIC is achieved through simulating the difference in the present value of future reinforcement with and without the injection, while LRMC charge is calculated through a single function representing three partial differentiations initiated by the nodal injection. If the LRIC/LRMC cost function is not steep with respect to the circuit's utilization, the difference between LRMC and LRIC charges should be very small.

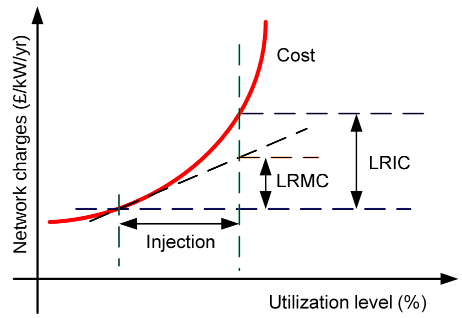
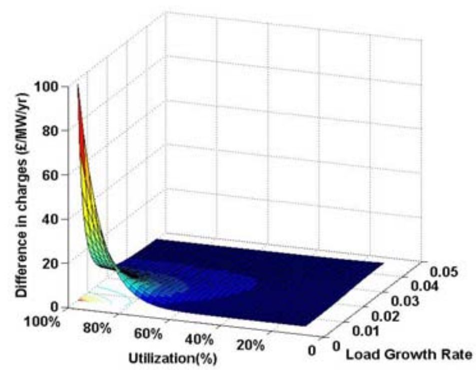
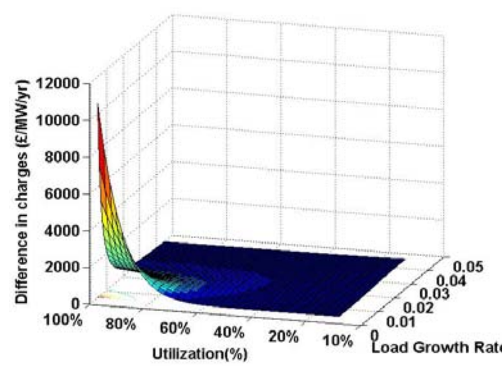


Fig. 3. Different calculation concepts of LRIC and LRMC

Two three-dimensional graphs in Fig. 4 demonstrate the difference in charges from the two approaches under different LGRs and at different circuits' loading levels. As seen from Fig. 4, the large difference is seen when the LGR is lower than 0.01 and the utilization is higher than 70%.



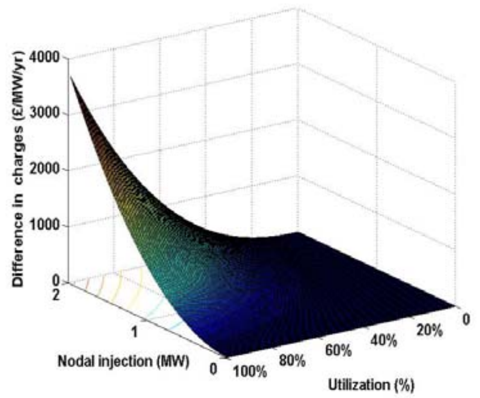
(a) 0.1MW injection case



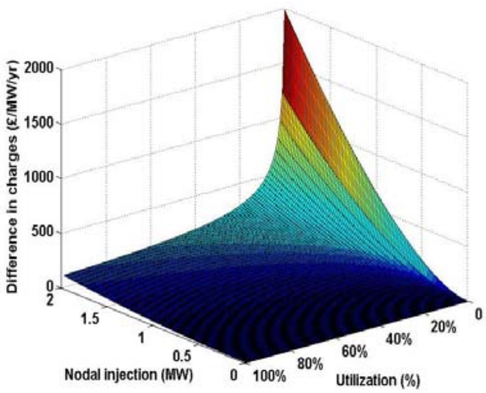
(b) 1MW injection case

Fig. 4. Difference in charges from the two methods

Two graphs in Fig. 5 show the difference in charges by varying the size of the nodal injection and the level of circuit utilization levels under 1.5% and 5% LGRs. Fig. 5.a shows that in the case of 1.5% LGR, the size of the nodal injection for LRIC has little influence on the difference when the circuit utilization is low, especially if the injection is smaller than 0.5MW. However, the difference grows apparent with the increasing nodal injection when the circuit utilization is high. It is due to the fact that a big nodal injection will greatly bring forward time to reinforce the circuit. In the high LGR case given in Fig. 5.b, the big difference only appears when the nodal injection is greater than about 0.5MW and the utilization is low. It is due to the steep slope of the LRMC cost function with respect to the circuit's loading level given in Fig. 3.



(a) 1.5% load growth rate case



(b) 5% load growth rate case

Fig. 5. Difference in charges from the two methods

B. Demonstration on a Practical System

In this section, the comparison of LRIC and LRMC pricing methods is carried out on a practical Grid Supply Point (GSP) area given in Fig. 6.

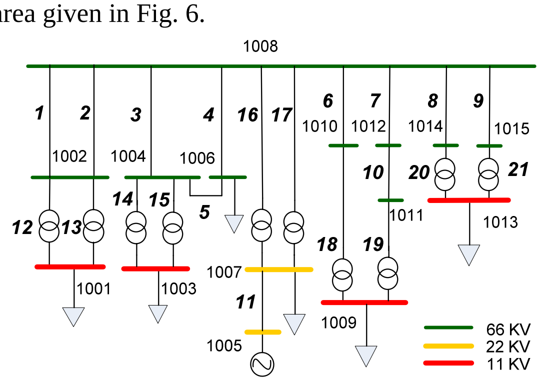


Fig. 6. A GSP area test system

The rationale in comparing the two methods on a practical system is that a nodal increment is likely to impact many circuits in the network. The difference between the two methods for each circuit might be modest, but accumulating these differences over all supporting circuits for a node could potentially produces large difference. The comparison is carried out under two conditions: i) two underlying LGRs: 1% and 5%; ii) two loading levels: base loading level and scaled-up level (by 20%). An injection of 1MW is employed for LRIC model. The comparisons are in terms of nodal LRIC and LRMC charges and tariffs.

For this practical system, if LRIC is adopted, it takes a computer 157 milliseconds to calculate the nodal charges for every single node in the network. But for LRMC, it only takes 51milliseconds on the same computer - 1/3 of the computational effort of the LRIC. For a large-scale system with 2000 nodes, it takes the computer 12 seconds to calculate LRIC charge for a single node and approximately 6 hours and 40minutes in total. In contrast, it takes only 0.5 second to compute LRMC charges for a single node and takes barely 17 minutes in total.

(1) Base case – base loading level

Table I gives nodal charges from LRIC and LRMC approaches under the base loading level. To assist the analysis, Fig. 7 depicts the utilization levels of branches in the base loading case. As seen from it, the most heavily loaded circuit is line No. 4 linking bus 1008 and bus 1006. Transformers 12-17 also have high loading levels.

TABLE I
COMPARISON OF CHARGES UNDER TWO LOAD GROWTH RATES (£/KW/YR)

Bus No.	LGR=1%			LGR=5%		
	LRIC	LRMC	Diff.	LRIC	LRMC	Diff.
1001	4.265	3.82	0.444	5.886	5.84	0.042
1002	0.607	0.546	0.061	4.419	4.39	0.03
1003	20.21	19.06	1.149	10.14	10.10	0.049
1004	18.61	17.61	1.001	9.04	8.997	0.04
1005	1.963	1.75	0.211	1.285	1.275	0.01
1006	18.16	17.18	0.979	6.698	6.66	0.039
1007	1.963	1.752	0.211	1.285	1.275	0.01
1009	0.122	0.097	0.025	10.16	10.02	0.143
1010	0.025	0.019	0.006	6.116	5.974	0.142
1011	0.245	0.16	0.085	12.94	12.61	0.329
1012	0.241	0.157	0.084	11.43	11.14	0.292
1013	0	0	0	2.053	1.961	0.092
1014	0	0	0	1.242	1.15	0.092
1015	0	0	0	2.3	2.121	0.179

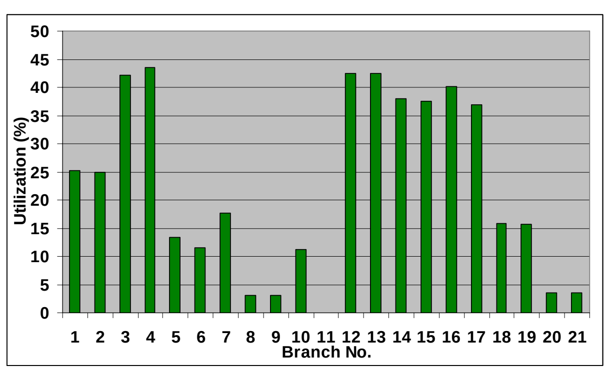


Fig. 7. Circuit utilization in base loading level case

When LGR is at 1%, the differences in charges from the two approaches are large for nodes 1001-1007, as they are supported by relatively highly utilized circuits. Also can be observed is that nodes 1009-1015 are supported by lightly loaded circuits, and correspondingly their charges are close to 0. In the 5% LGR case, the charges at nodes 1009-1015 become significantly larger because when the underlying LGR is higher, the time to reinforce network assets is nearer and therefore a nodal injection would have a greater impact on the present value of future investment. In comparison, nodes 1003-1006 are supported by heavily utilized circuits, their charges decrease as the LGR increases. Generally, the conclusions from the simple example are still applicable here: cases with small LGRs and high loading levels see big differences in LRIC and LRMC charges. It is also true for cases with large LGRs and low loading levels.

Two most commonly used revenue reconciliation approaches-fixed adder and fixed multiplier are employed here to demonstrate the degree of adjustments required to meet the target revenue, their relative merits and impacts on LRIC and LRMC charges. The tariffs are given in tables II and III.

TABLE II
COMPARISON OF TARIFFS USING FIXED ADDER METHOD (£/KW/YR)

Bus No.	LGR=1%		LGR=5%	
	LRIC	LRMC	LRIC	LRMC
1001	6.659	6.806	11.073	11.073
1002	3.001	3.532	9.606	9.623
1003	22.604	22.046	15.327	15.333
1004	21.004	20.596	14.227	14.230
1005	4.357	4.736	6.472	6.508
1006	20.554	20.166	11.885	11.893
1007	4.357	4.738	6.472	6.508
1009	2.516	3.083	15.347	15.253
1010	2.419	3.005	11.303	11.207
1011	2.639	3.146	18.127	17.843
1012	2.635	3.143	16.617	16.373
1013	2.394	2.986	7.240	7.194
1014	2.394	2.986	6.429	6.383
1015	2.394	2.986	7.487	7.354

From table II, when LGR is 1%, the largest difference in LRIC and LRMC tariffs is 0.592£/kW/yr for nodes 1013-1015. It is because that although these nodes have zero charges, fixed adder allocates the under-recovered revenue equally to all network nodes, thus resulting in the fixed adder of £2.394/kW/yr for LRIC and £2.986/kW/yr for LRMC. When LGR increases to 5%, the largest difference decreases to 0.284£/kW/yr (for node 1011). For all other nodes, the charges from the LRIC and LRMC approaches yield quite similar tariffs. Compared with 1% LGR case, tariffs for this case are much higher, because that when loads grow faster, time to reinforce circuits will be nearer, leading to high charges. From the table, it can also be seen that the fixed adder approach maintains the relative differences in nodal tariffs the same as the nodal charges, therefore minimizing the potential distortion to the economic charges.

As for the fixed multiplier method, it amplifies the relative difference of nodal charges, as a result, higher charges getting even higher tariff and 0 charges remaining 0, as shown in table III. For the low LGR case, the biggest difference in LRIC and LRMC tariffs is 0.357 £/kW/yr for node 1004, which has been reduced from the original difference of 1.001£/kW/yr in charges, as LRIC and LRMC methods see different multipliers, 0.25 for LRIC and 0.34 for LRMC. When it comes to the high LGR case, the tariffs reconciled from LRIC and LRMC charges are quite close and the biggest difference is for node 1011, counted as 0.433£/kW/yr. Compared with the difference of 0.329£/kW/yr in charges (in table I), this tariff difference is amplified by the multiplier. Potentially, if there are few excessively high nodal charges, a modest multiplier would lead to extremely high tariffs for the few nodes.

TABLE III
COMPARISON OF TARIFFS USING FIXED MULTIPLIER METHOD (£/KW/YR)

Bus No.	LGR=1%		LGR=5%	
	LRIC	LRMC	LRIC	LRMC
1001	5.342	5.134	10.600	10.592
1002	0.760	0.734	7.958	7.962
1003	25.315	25.617	18.261	18.318
1004	23.311	23.668	16.280	16.318
1005	2.459	2.352	2.314	2.312
1006	22.747	23.090	12.062	12.079
1007	2.459	2.355	2.314	2.312
1009	0.153	0.130	18.297	18.173
1010	0.031	0.026	11.014	10.835
1011	0.307	0.215	23.303	22.871
1012	0.302	0.211	20.584	20.204
1013	0.000	0.000	3.697	3.557
1014	0.000	0.000	2.237	2.086
1015	0.000	0.000	4.142	3.847

(2) Higher loading level – 20% scaling up

In this part, all loads are scaled up by 20%, thus increasing all circuits' utilization by approximately 20%. The scaled up loading levels of all branches are given in Fig. 8.

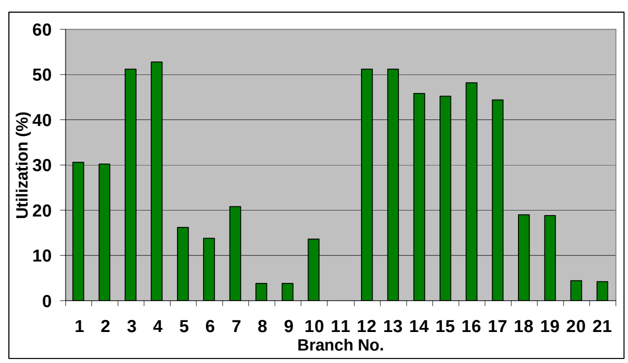


Fig. 8. Circuit utilization in scaling loading level case

Table IV summarizes the charges from the two charging approaches for the two LGR cases. Obviously, charges follow the same patterns as the base case, but they are much higher because of the increased circuit utilization levels. Compared with results given by table I, the increments in charges are similar for both approaches, where the lower LGR sees greater increments in charges and the high LGR sees small increments.

Tables V provides tariffs calculated using fixed adder method. In the low LGR case, the fixed adder approach gives negative tariffs for some nodes. It is due to that charges are dominated by high charges at buses 1003, 1004 and 1006, which are supported by highly utilized circuits. The revenue recovered from these three nodes alone already exceeds the allowed revenue. Consequently, a negative adder is obtained, leading to negative tariffs for the majority of the nodes in the system. When the LGR rises up to 5%, tariffs for all nodes are positive because of the positive adder and the difference in tariffs becomes small compared with the 1% LGR case.

TABLE IV
COMPARISON OF CHARGES UNDER TWO LOAD GROWTH RATES (£/KW/YR)

Bus No.	LGR=1%			LGR=5%		
	LRIC	LRMC	Diff.	LRIC	LRMC	Diff.
1001	12.52	11.43	1.087	6.29	6.25	0.037
1002	1.757	1.61	0.146	4.70	4.68	0.026
1003	60.19	57.35	2.836	10.87	10.83	0.044
1004	55.21	52.76	2.451	9.66	9.62	0.036
1005	5.39	4.894	0.496	1.38	1.38	0.008
1006	53.87	51.47	2.398	7.16	7.12	0.035
1007	5.39	4.89	0.496	1.38	1.36	0.008
1009	0.39	0.32	0.068	11.21	11.08	0.134
1010	0.076	0.06	0.014	6.57	6.45	0.125
1011	0.78	0.54	0.237	14.45	14.14	0.314
1012	0.77	0.53	0.233	12.85	12.56	0.282
1013	0	0	0.000	2.18	2.1	0.082
1014	0	0	0.000	1.31	1.23	0.083
1015	0	0	0.000	2.43	2.27	0.162

TABLE V
COMPARISON OF TARIFFS USING FIXED ADDER METHOD (£/KW/YR)

Bus No.	LGR=1%		LGR=5%	
	LRIC	LRMC	LRIC	LRMC
1001	-5.196	-4.834	9.036	9.042
1002	-15.959	-14.654	7.446	7.472
1003	42.474	41.086	13.616	13.622
1004	37.494	36.496	12.406	12.412
1005	-12.326	-11.370	4.126	4.172
1006	36.154	35.206	9.906	9.912
1007	-12.326	-11.374	4.126	4.152
1009	-17.326	-15.944	13.956	13.872
1010	-17.640	-16.204	9.316	9.242
1011	-16.936	-15.724	17.196	16.932
1012	-16.946	-15.734	15.596	15.352
1013	-17.716	-16.264	4.926	4.892
1014	-17.716	-16.264	4.056	4.022
1015	-17.716	-16.264	5.176	5.062

TABLE VI
COMPARISON OF TARIFFS USING FIXED MULTIPLIER METHOD (£/KW/YR)

Bus No.	LGR=1%		LGR=5%	
	LRIC	LRMC	LRIC	LRMC
1001	4.436	4.276	8.767	8.769
1002	0.622	0.602	6.551	6.566
1003	21.325	21.454	15.150	15.194
1004	19.560	19.737	13.464	13.497
1005	1.910	1.831	1.923	1.936
1006	19.086	19.254	9.979	9.989
1007	1.910	1.829	1.923	1.908
1009	0.138	0.120	15.624	15.545
1010	0.027	0.022	9.157	9.049
1011	0.276	0.202	20.140	19.838
1012	0.273	0.198	17.910	17.621
1013	0.000	0.000	3.038	2.946
1014	0.000	0.000	1.826	1.726
1015	0.000	0.000	3.387	3.185

As for the tariffs from the fixed multiplier method given by table VI, compared with the base case results in table III, they become a little bit smaller for all nodes because of the increased demand. However, compared with the tariffs calculated with the fixed adder approach, there is no negative tariff obtained in the 1% LGR case. On the other hand, all tariffs in this case are smaller than the charges provided in table IV as a smaller fixed multiplier scales down all charges proportionally.

The revenue reconciliation mechanism used by a DNO is very important as it decides how LRIC or LRMC charges should be shaped into tariffs seen by network users. In practice, a large proportion of DNOs' revenue may be recovered through the reconciliation mechanism. The fixed adder approach can maintain the same level of differentiation between nodal tariffs, thus minimizing any distortion over the pure incremental/marginal costs. In contrast, the fixed multiplier approach maintains the relativity between nodal tariffs, but the relativity is proportionally amplified by the same level. This could be considered as the distortion of the cost signals that network customers would see. The fixed adder approach is thus preferred by the majority of DNOs in the UK.

VI. DISCUSSIONS

Generally, the difference in charges and tariffs from LRIC and LRMC approaches is affected by three major factors: the circuit's utilization level, LGR and the size of nodal injection. For the majority of the operating conditions in practice, they would yield very similar results. LRMC is a good approximation to LRIC except for few extreme cases, where LRIC should be used to better reflect the extent of the impacts on the network imposed by a nodal power increment. Additional benefit with LRMC is that the interim results from it can provide further insights into how different factors, such as how the circuit loading level and LGR would impact on the long-term development costs and to what extent they would impact. Such information is not readily available from the LRIC charging model.

It should be noted that locational charges set by either LRIC or LRMC are to recover the network fixed costs. This is of paramount importance to DNOs at the moment when they are expecting to connect substantial amount of Distribution Generators (DGs). Efficient locational messages will incentivise the prospective DGs to connect to appropriate sites so as to minimise the network development costs.

The core of the LRIC charging model proposed in [15] has been adopted by three of the UK's major distributors, Western Power Distribution (WPD, UK) Électricité de France (EDF) and CE Electric.

The long-run marginal and incremental cost pricing models provide locational messages to minimise the network development costs. The short-run and long-run pricing should be complementary and interactive. The short-run locational marginal pricing aims to minimise congestion and losses, thus improving the efficiency of the existing network and delaying the need network upgrades. Efficient long-run messages should encourage prospective network customers to better utilize the existing network, thus reducing congestion and losses in the long run. Network operators should strike the right balance between network investment costs and network congestion and losses costs, which should be reflected in the interaction between the long-run and short-run pricing.

VII. Conclusions

In this paper, a novel LRMC charging method based on analytical approach is proposed, which directly relates the nodal power increment to the change in the present value of future network investment. Results on the two systems using the proposed method are compared and contrasted with those from the LRIC approach. Based on the extensive analysis, the following key findings can be concluded:

(i) In terms of accuracy, the LRIC and LRMC approaches yield quite similar results when the size of the nodal injection for LRIC is small. The biggest difference appears when circuits are highly loaded and load growth rate (LGR) is small. When the injection becomes large, the discrepancies between the two approaches become apparent and the biggest difference shows up when circuits are lightly loaded and LGR is high. As for tariffs, they are highly dependant on charges, and largely follow the same pattern as for the charges.

(ii) In terms of speed, the LRIC needs to run power flow analysis twice for each nodal injection in order to examine the effects of an injection on the long-term development costs. For a large system, the computational burden grows exponentially with the increase in the size of the network. The proposed LRMC, on the other hand, saves significant computational time for large-scale networks by utilizing sensitivity analysis, avoiding running power flow analysis for every single nodal injection.

(iii) In terms of flexibility, the LRIC model, working through simulation approach, can examine the impacts imposed on a network by any size of injection. But, the proposed LRMC can only accurately represent a very small change. For a large size of injection, the charges obtained with LRMC can deviate from those calculated with the LRIC.

(iv) Finally, revenue reconciliation process is very important in how it might shape the relative difference in LRIC and LRMC charges. The fixed adder approach uniformly scales up/down all nodal charges, hence preserving the absolute difference in nodal charges. The fixed multiplier, on the other hand, amplifies the nodal relativity. If the amplification becomes significant, it could considerably distort the impact that a nodal power injection might have on the network development cost. As a consequence, the industry in general favors the fixed adder approach over the fixed multiplier.

To summarize, the proposed LRMC charging model produces very similar results with that of LRIC for the majority of operating conditions. It is a good supplement to LRIC method not only because of its computational efficiency but also because of the additional insights that the interim results offer for understanding the charging problems and the consequential charges.

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IX. Biographies

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